

Black Holes as Fast Scramblers

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Abstract

Within the framework of the information paradox of black holes, one avenue of inquiry brings us to the effects on information and whether it is truly lost forever when past the event horizon.

Thermalization of quantum information over the event horizon can be revealed quickly in the form of Hawking radiation. Rapid thermalization, or fast scrambling, is a theory readily explained by quantum information theory. By understanding the formalism of quantum mechanics and utilizing random unitary transformations on entangled subsystems, we can explain how during the latter half of the Page curve—for when half of the initial entropy of the black hole has dissipated—allows for information to be instantly retrievable. [1][2]

The Complementarity

Alice has message M jumps into black hole B . Bob observes the outgoing Hawking radiation E

Kruskal coordinates will be the means to describe the difference in spacetime between Alice and Bob, once they pass the event horizon of the black hole.

The tangent plane of the horizon is $X^\pm = \pm \rho e^{\pm \omega}$, with ρ is the proper radial distance from the horizon and ω is the dimensionless Rindler time.

Singularity $X^+ X^- = R^2$

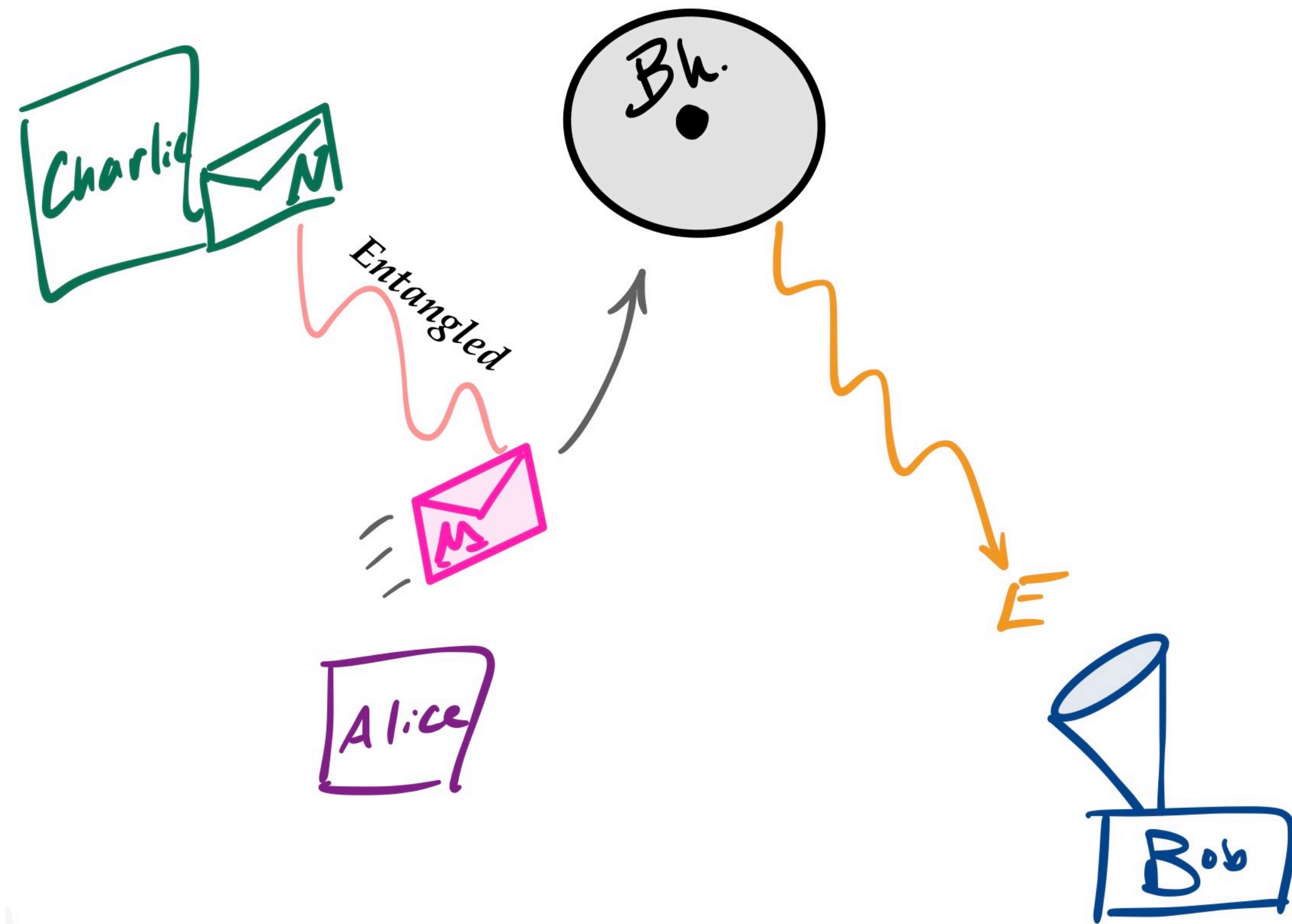


Figure 1 (above): First, Charlie's N is maximally entangled with Alice's M . As M is tossed into the black hole, Bob's decoder is taking in emitted radiation from the black hole's event horizon.

Figure 2 (right): Then, maximally entangled with N , M also becomes thermalized (see section on **Thermalization Time** for scrambling time) over the event horizon of the black hole becoming maximally entangled with the contents of the black hole.

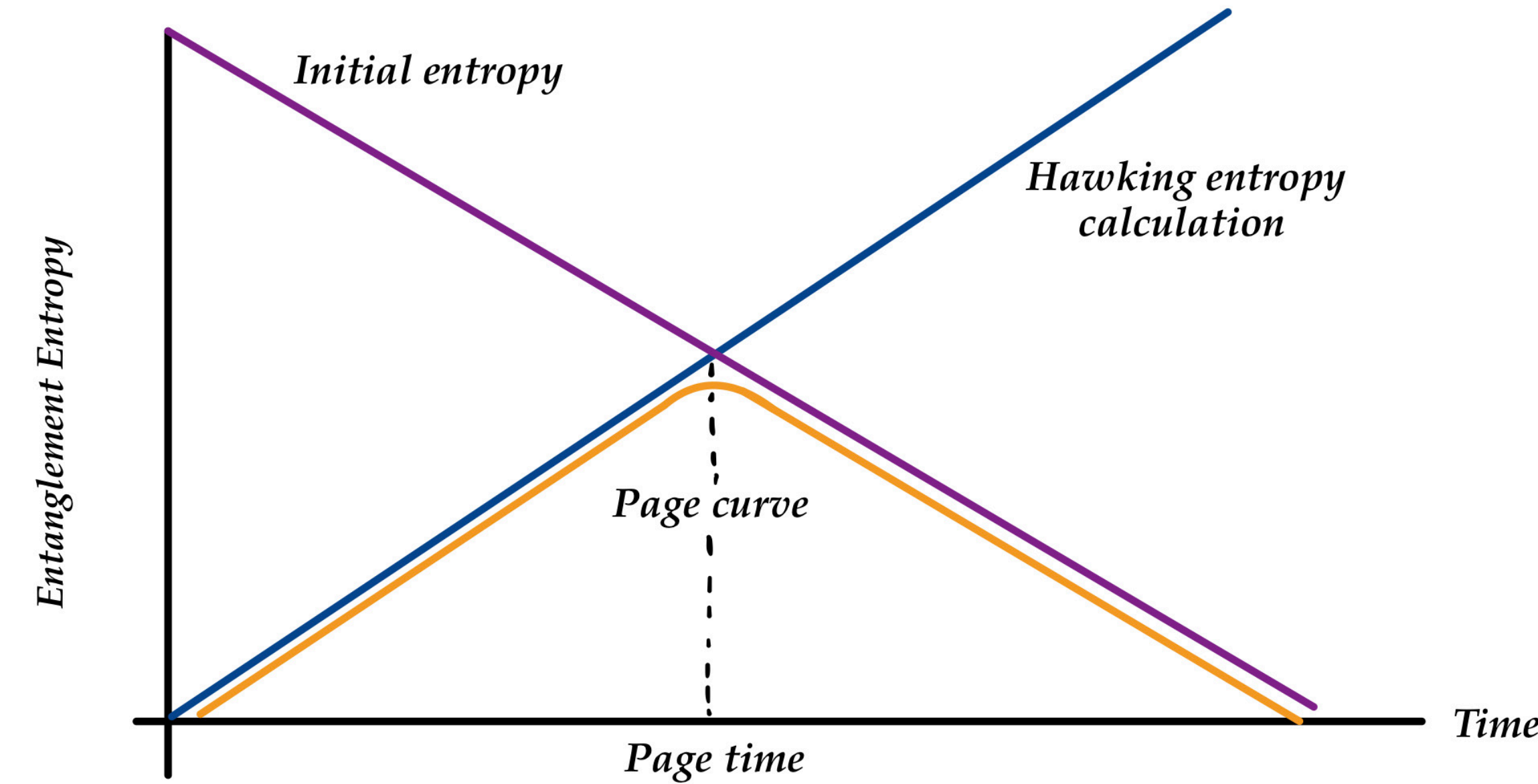


Figure 5: The Page Curve constructed to show the when the initial entropy of the black hole matches the entropy calculation from Hawking to signify the Page time, which marks the half-way point of the entropic life-span of the black hole. [5] [6]

Quantum Randomization/Scrambling

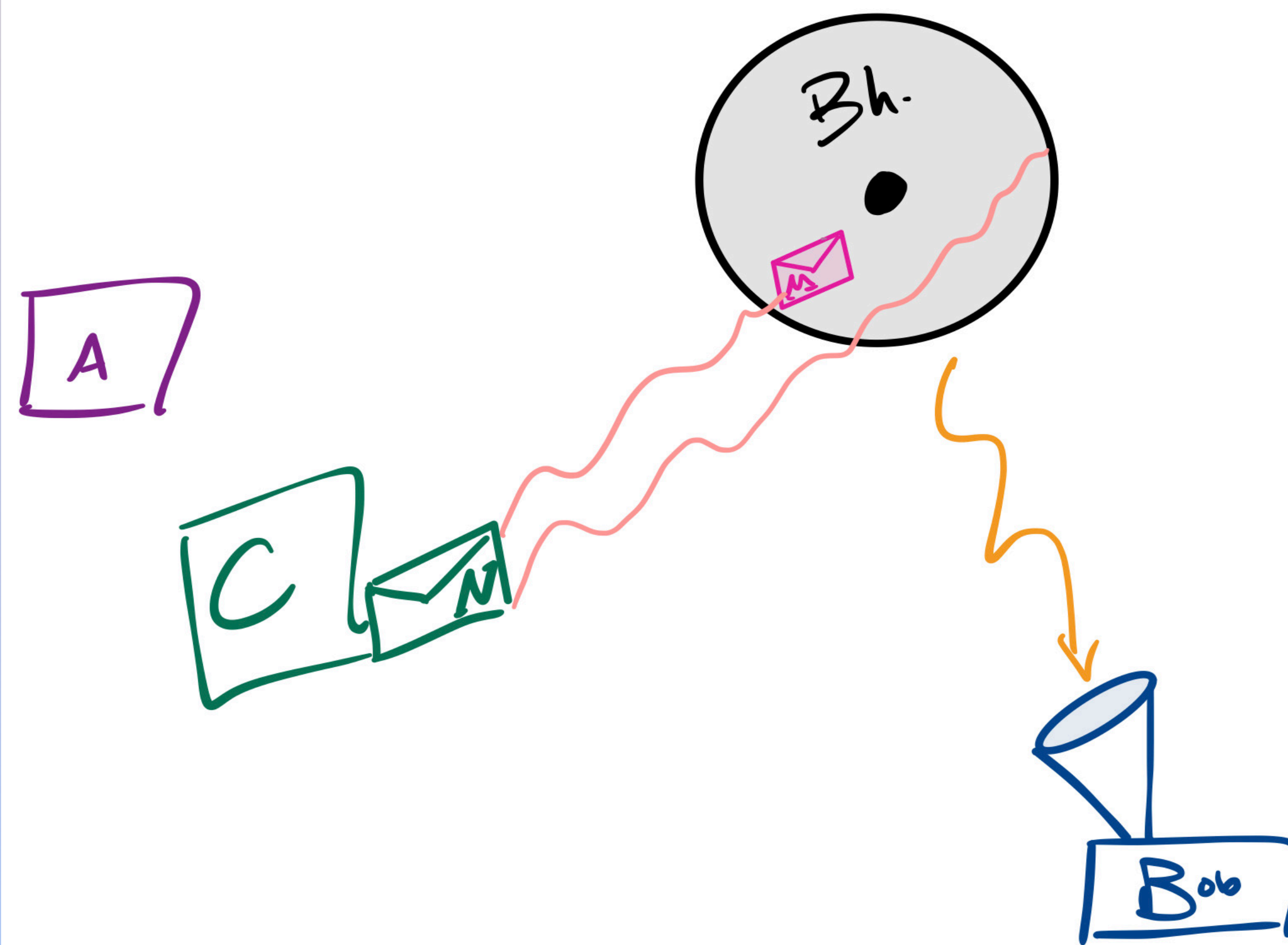
Quantum Bits:(i.e. spin- $\frac{1}{2}$ particles) $\{\uparrow, \downarrow\}$ have two possibilities for any string of bits with length k . Thus, dimensions of Alice's memory equates to

$$|M| = \binom{2}{1} \binom{2}{1} \dots (k \text{ times}) \dots \binom{2}{1} = 2^k$$

An entangled quantum system $|\Phi\rangle^{MN}$ involving two subsystems $|a\rangle^M$ and $|a\rangle^N$

$$|\Phi\rangle^{MN} = \frac{1}{\sqrt{|M|}} \sum_{a=1}^{|M|} |a\rangle^M \otimes |a\rangle^N \quad (1)$$

Density operators define the system, it encodes all the accessible information about a quantum mechanical system. They will allow us to manipulate observables using unitary transformations—which preserve reversibility while still randomizing the information of M



Thermalization Time

There exists ties with fast scramblers and the membrane paradigm under thermalization time. At the level of the Schwarzschild radius r_s the stretched horizon's Ohmic behavior dissipates a point charge in Rindler time like [3][4]

$$\omega = \log r_s$$

This is exactly how much time is needed for quantum-bits of information diffuse over the stretched horizon.[2]

Retrieval

We have to make the assumption, from Don Page's work, that until half of the entanglement entropy has emitted has Hawking radiation, the black hole will not be maximally entangled with its surroundings.[1] Page Time marks the point where Alice's message may be entered, scrambled, and retrieved by Bob with incredible speed.

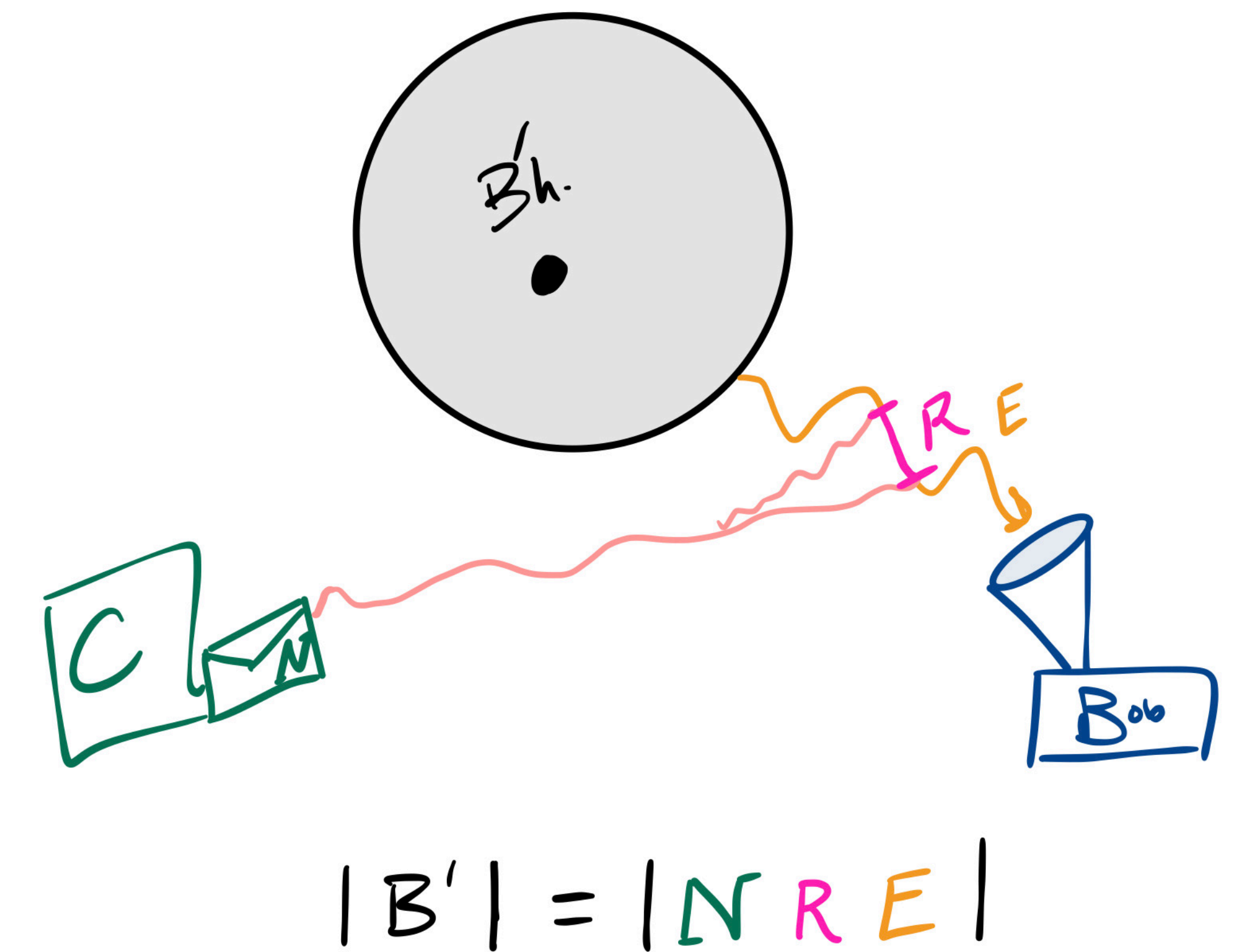
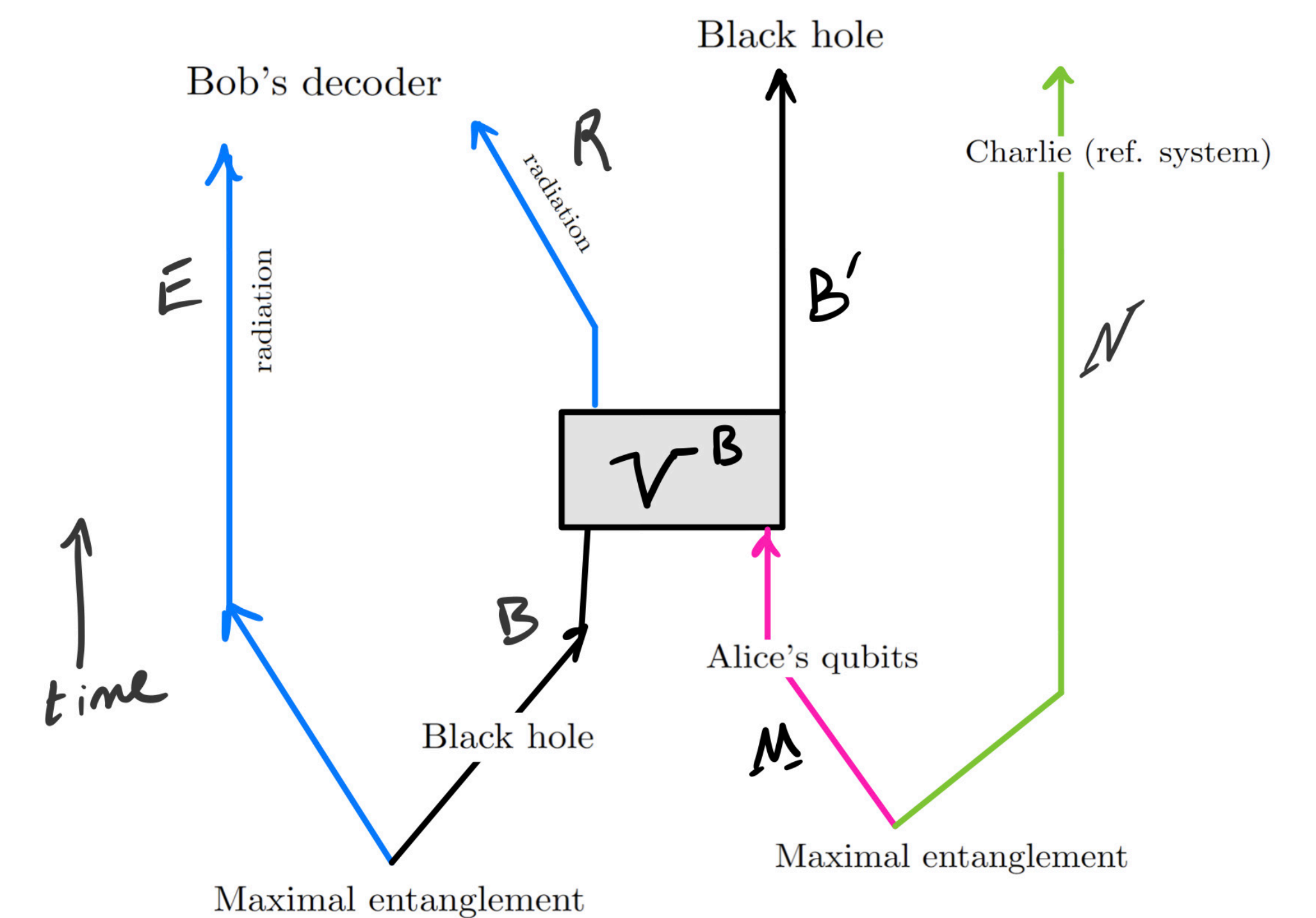


Figure 3(above): After the message has been scrambled, it emerges as R among E . Bob is then able to take the message and decode from there.

Figure 4(below): Plotting the events in the experiment [1]



References

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